
Jet and Rocket Propulsion

AE4451

LECTURE 13

Overview

what we saw last time:

- ramjets
 - ideal cycle analysis
 - new expressions for performance metrics: specific thrust

today:

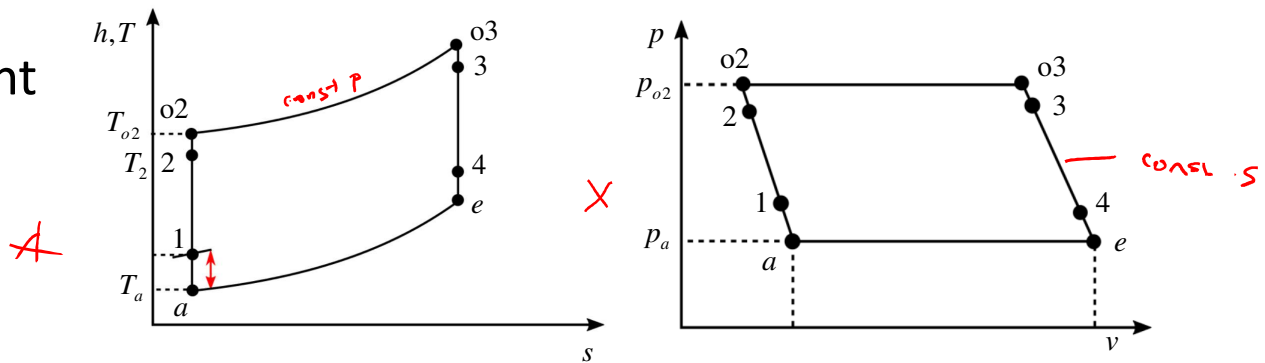
- real ramjet cycle analysis for different components

Ramjets

Real ramjet performance

We want to remove some of the idealizations in the previous analysis:

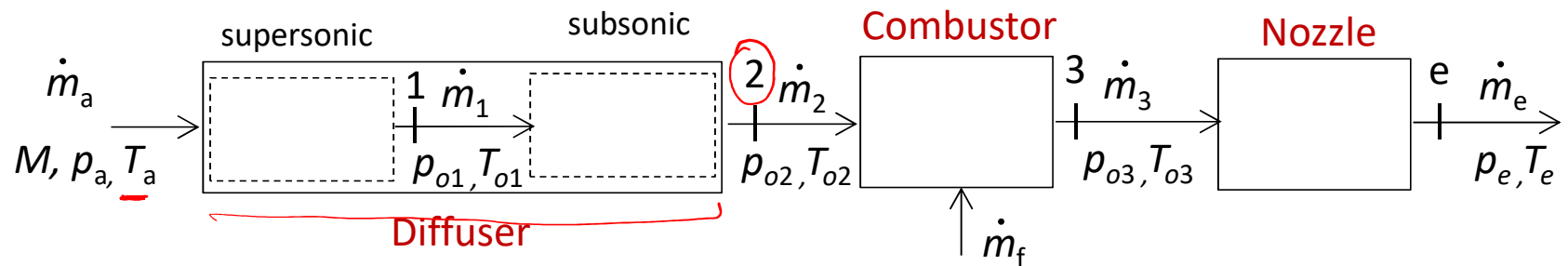
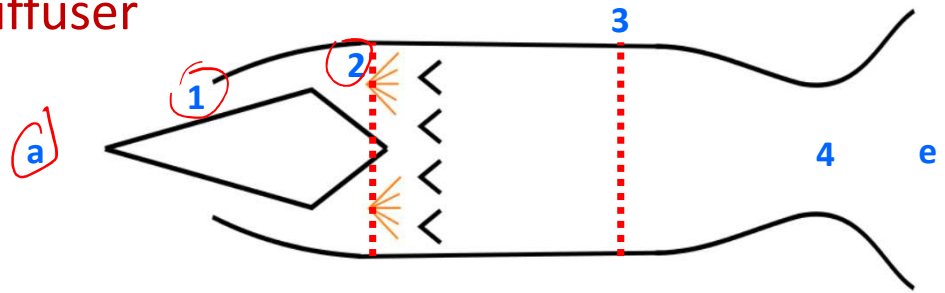
- inlet/diffuser, combustor and nozzle are no longer reversible
 - pressure losses will result
- combustor does not achieve ideal heat release
- some of the fuel is unburned and/or the combustion is “incomplete” (e.g., CO instead of CO₂)
- nozzle not perfectly expanded
- some idealizations maintained:
 - no heat losses, $c_p = \text{constant}$



Ramjets

Real ramjet performance: diffuser

use same p_a, T_a
 M and T_{o3} as in ideal case



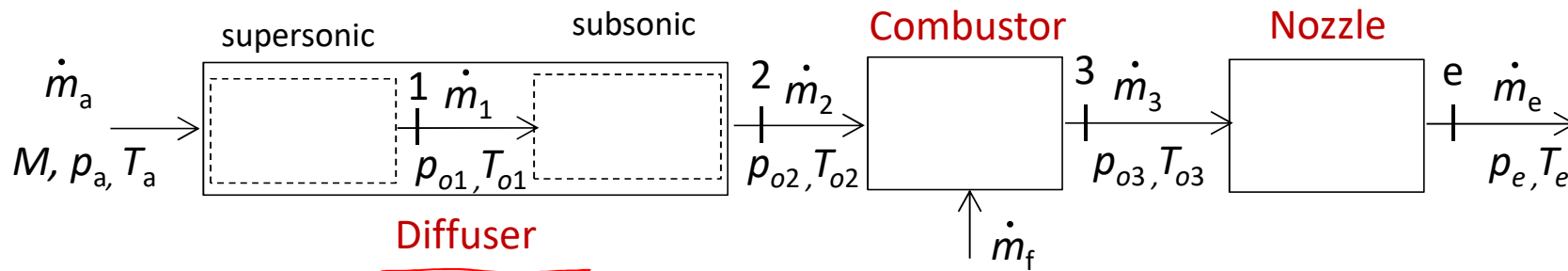
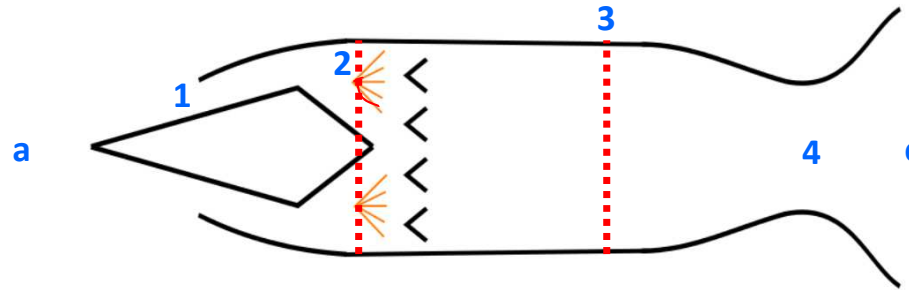
Mass $\dot{m}_a = \dot{m}_1 = \dot{m}_2$ recall $\sum \dot{m}_{outlets} = \sum \dot{m}_{inlets}$

Energy $\dot{m}_a h_{oa} = \dot{m}_a h_{o1} = \dot{m}_a h_{o2}$ recall $\dot{Q}_{in} - \dot{W}_{shaft} = \frac{d}{dt} \int_{CV} \rho e_o dV + \int_{CS} \rho h_o (\vec{u} \cdot \hat{n}) dA$ assumption is for no heat in

replacing enthalpy with T $\Rightarrow T_{oa} = T_{o1} = T_{o2} \Rightarrow T_{o2} = T_a \left(1 + \frac{\gamma-1}{2} M^2 \right)$ $T_{oa} = T_a \left(1 + \frac{\gamma-1}{2} M^2 \right) \times$

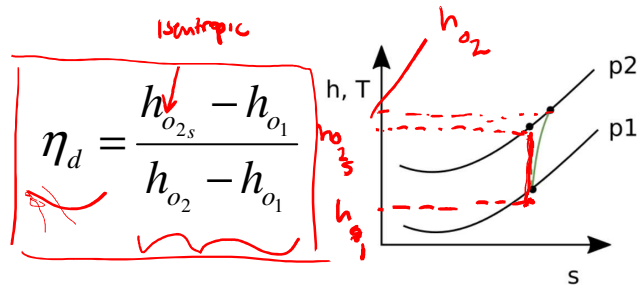
Ramjets

Real ramjet performance



Entropy to get stagnation pressure, use adiabatic efficiency to compare to isentropic case

recall adiabatic efficiency form for compressor

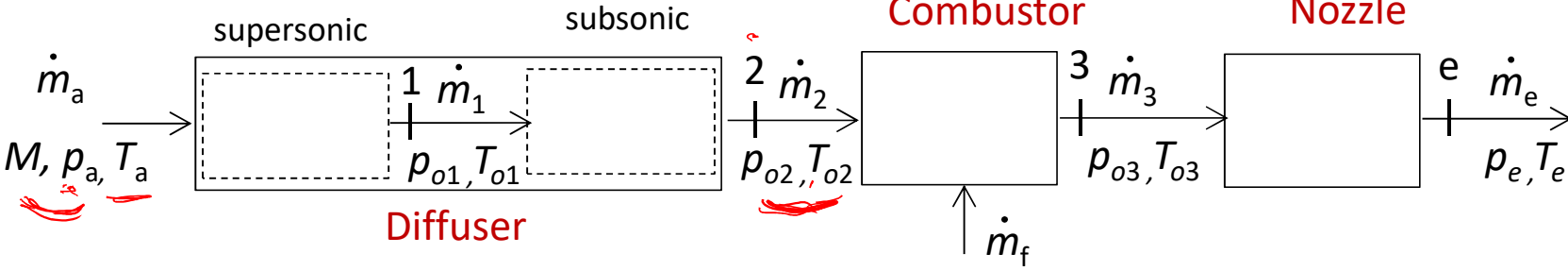


$$\eta_d = \frac{h_{o2s} - h_{o1}}{h_{o2} - h_{o1}}$$

i.e. real work/isentropic work
or
isentropic enthalpy change/real enthalpy change

Ramjets

Real ramjet performance: diffuser



Entropy

$$\eta_d = \frac{T_{o2s} - T_{oa}}{T_{o2} - T_{oa}} \Rightarrow \left(\frac{T_{o2s}}{T_{oa}} \right) = \eta_d \left(\frac{T_{o2}}{T_{oa}} - 1 \right) + 1 \quad \text{recall} \quad \frac{P_0}{P} = \left(\frac{T_0}{T} \right)^{\frac{\gamma}{\gamma-1}}$$

$$\Rightarrow \frac{p_{o2s}}{p_a} = \left[\eta_d \left(\frac{T_{o2s}}{T_a} - 1 \right) + 1 \right]^{\frac{\gamma}{\gamma-1}}$$

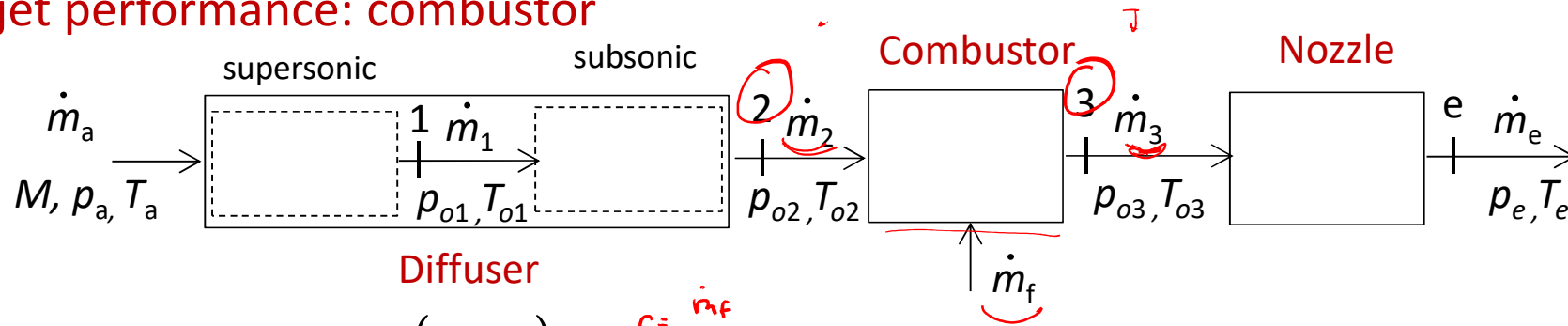
modified by **ram recovery factor** ($r_d(M) \leq 1$) to account for shock losses

$$\Rightarrow \frac{p_{o2}}{p_a} = r_d \left\{ 1 + \eta_d \left(\frac{T_{o2}}{T_a} - 1 \right) \right\}^{\frac{\gamma}{\gamma-1}} \quad \text{and using substitution} \quad \frac{T_0}{T} = 1 + \frac{\gamma-1}{2} M^2$$

$$\Rightarrow \frac{p_{o2}}{p_a} = r_d \left\{ 1 + \eta_d \left(\frac{\gamma-1}{2} M^2 \right) \right\}^{\frac{\gamma}{\gamma-1}}$$

Ramjets

Real ramjet performance: combustor



Mass $\dot{m}_a + \dot{m}_f = \dot{m}_3 = \dot{m}_a (1 + f)$

$f = \frac{\dot{m}_f}{\dot{m}_a}$

Energy recall $\dot{m}h_{o,in} = \dot{m}h_{o,exit} + \dot{Q}_{loss}$
 $\dot{Q}_{loss} = \dot{m}(h_{o,in} - h_{o,exit}) = \dot{m}\Delta h_R$

recall $f = \frac{T_{o3} - T_{o2}}{\Delta h_R / c_p - T_{o3}}$

combustion efficiency: how much available chemical energy is converted to thermal energy

$f = \frac{T_{o3} - T_{o2}}{\eta_b \Delta h_R / c_p - T_{o3}}$

can now introduce the pressure burner ratio (< 1);
 i.e. pressure drops at exit of burner

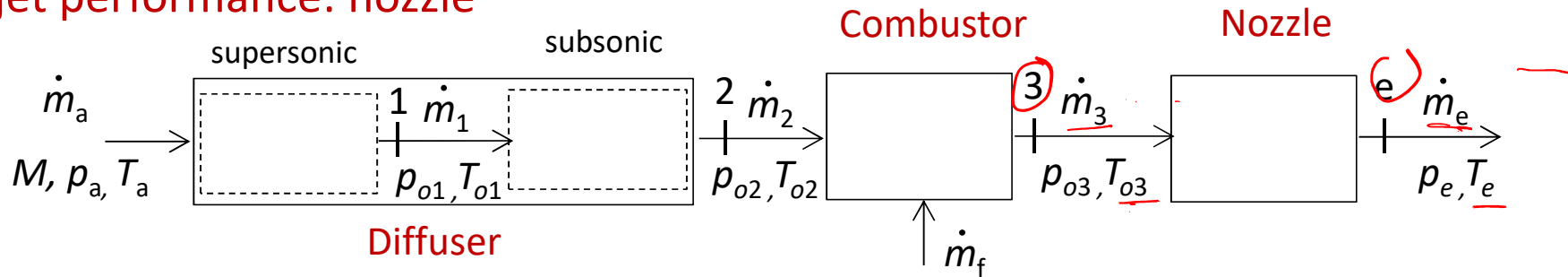
$\frac{p_{o3}}{p_{o2}} = p_{rb}$

$f = \frac{T_{o3}/T_{o2} - 1}{(\eta_b \Delta h_R / c_p T_{o2}) - T_{o3}/T_{o2}}$

can only use efficiency to describe this if dealing with expansion/compression process

Ramjets

Real ramjet performance: nozzle



Mass $\dot{m}_3 = \dot{m}_e$

Energy $\dot{m}_3 h_{o3} = \dot{m}_e h_{oe} \Rightarrow h_{o3} = h_{oe} = h_e + \frac{u_e^2}{2}$

$\Rightarrow u_e = [2(h_{o3} - h_e)]^{1/2} = [2c_p(T_{o3} - T_e)]^{1/2}$

isotropic

Entropy

$\eta_N = \frac{h_{o3} - h_e}{h_{o3} - h_{e_s}} = \frac{T_{o3} - T_e}{T_{o3} - T_{e_s}} \Rightarrow T_e = T_{o3} \{1 - \eta_N\} + \eta_N T_{e_s}$

$T_{e_s} = T_{o3} \left(\frac{p_e}{p_{o3}} \right)^{\gamma-1/\gamma}$ if isentropic

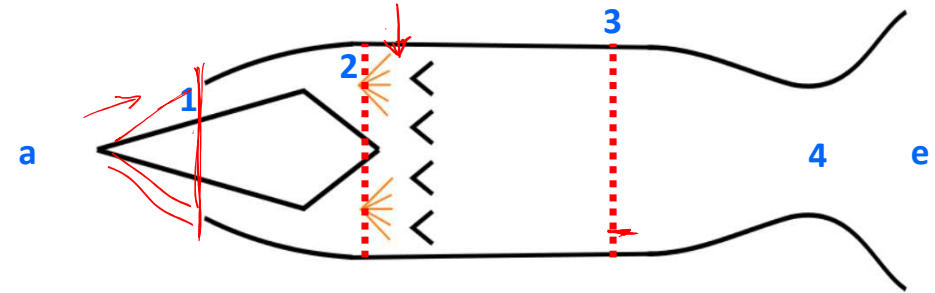
$T_e = T_{o3} \left\{ 1 - \eta_N \left[1 - \left(\frac{p_e}{p_{o3}} \right)^{\gamma-1/\gamma} \right] \right\}$

Ramjets

A closer look at performance: ideal ramjet

$$\eta_o = \frac{ST}{f} \frac{u}{\Delta h_R} = \frac{1}{TSFC} \frac{u}{\Delta h_R}$$

$$\eta_o = \frac{M \sqrt{\gamma R T_a} \left[(1+f) \sqrt{\frac{T_{o3}}{T_a \left(1 + \frac{\gamma-1}{2} M^2 \right)}} - 1 \right]}{\frac{T_{o3}/T_a - \left(1 + \frac{\gamma-1}{2} M^2 \right)}{\Delta h_R / c_p T_a - T_{o3}/T_a}} \frac{u}{\Delta h_R}$$



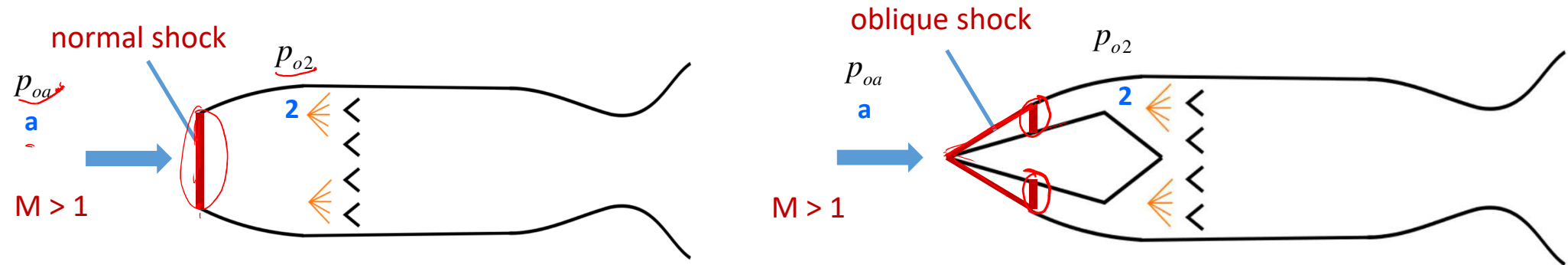
notice overall efficiency a function of:

- Mach number
- heat of reaction
- temperature ratio

- at higher M, efficiency starts to drop, very inefficient for M > 5
- combustion should occur at subsonic speeds
- entering air must be slowed; happens thanks to shock waves at inlet
- these shocks will result in important losses

Ramjets

A closer look at performance: ideal ramjet



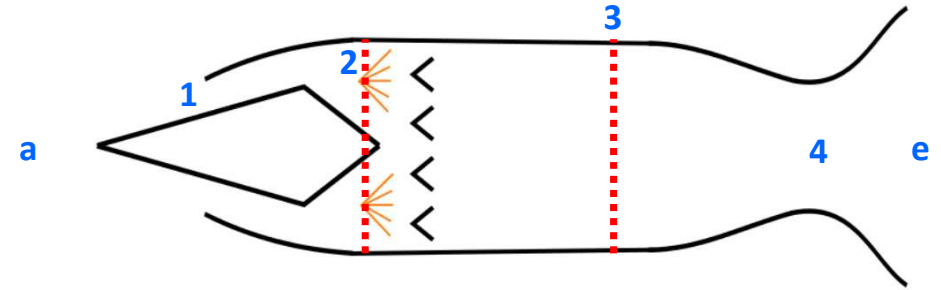
- ramjets use spikes to reduce shock pressure losses

$\frac{P_{o2}}{P_{oa}}$ is lower without spike due to area of normal shock, i.e. "lower stagnation pressure recovery" or "lower ram recovery"

⇒ spike improves ramjet efficiency

Ramjets

A closer look at performance: real ramjet



$$\eta_o = \frac{ST}{f} \frac{u}{\Delta h_R} = \frac{1}{TSFC} \frac{u}{\Delta h_R}$$

$$ST = \frac{\tau}{\dot{m}_a} = \left[(1+f)u_e - M \sqrt{\gamma R T_a} \right] + \frac{(p_e - p_a)A_e}{\dot{m}_a}$$

u depends on nozzle design

$$TSFC = \frac{f}{ST}$$

$$\eta_o = \frac{1}{TSFC} \frac{u}{\Delta h_R}$$

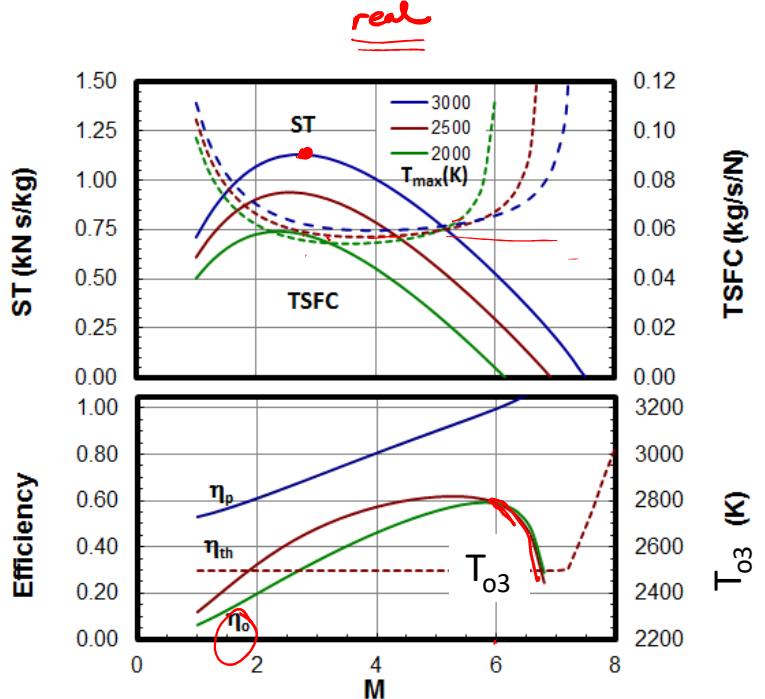
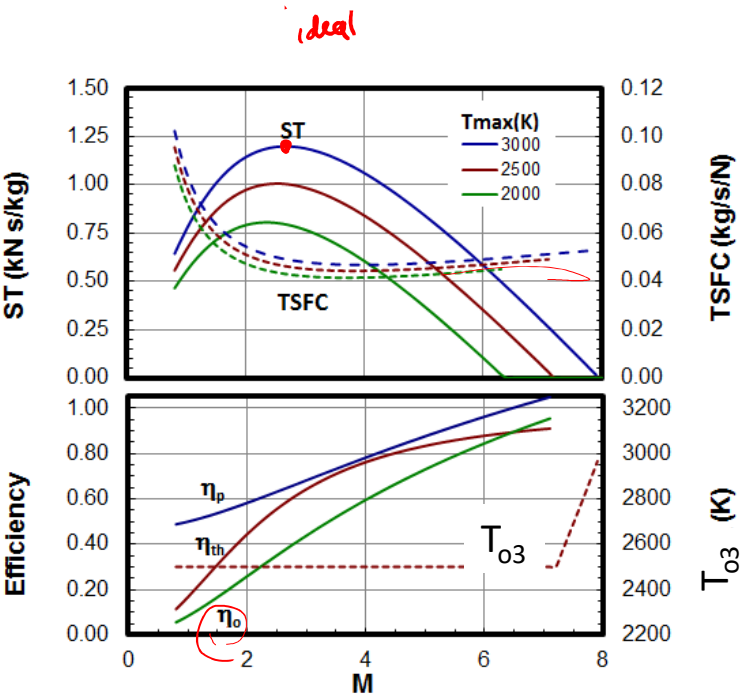
$$\eta_p = \frac{\tau u}{\Delta \dot{K}E} = 2 \frac{ST}{u \left[(1+f)(u_e/u)^2 - 1 \right]}$$

$$\eta_{th} = \frac{\eta_o}{\eta_p}$$

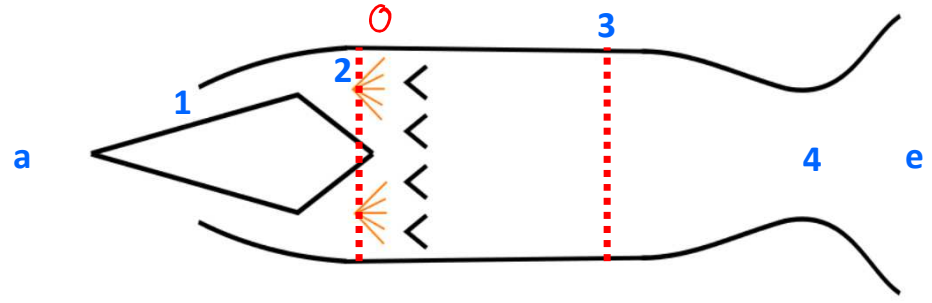
no change in these expressions compared to ideal case

Ramjets

Real vs ideal ramjet performance



- real ramjet has:
- lower specific thrust
 - higher specific fuel consumption
 - lower overall efficiency



$$\eta_{th} = \frac{\dot{\Delta KE}}{m_f \Delta h_R}$$

$$\eta_p = \frac{u}{\dot{\Delta KE}}$$

Ramjets

Scramjet = supersonic-combustion ramjet

NASA X-43: reached record speed of Mach 9.6

- melting of parts at high M avoided with water cooling from M > 3
- succeeded in 2006 by X-51 program



Boeing X-51A WaveRider (USAF graphic)
Mach 5.1

- 210 s powered flight time, 6 min total flight time
- longest hypersonic air-breathing flight



Hypersonic Air-breathing Weapons Concept (HAWC)
Mach 5

- scramjet-based hypersonic cruise missile
- first flight September 2021
- final successful test flight: end of Jan 2023